LOCAL SEISMIC TOMOGRAPHY AS INTERPRETATIVE MODEL OF THE EARTH STRUCTURE AND MULTI-CHANNEL INDICATOR OF MARINE SEDIMENTS: A BRIEF SUMMARY

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Abstract

Local earthquake tomography (LET) offers a valuable criterion for theoretically modelling the earth structure involved in seismic activity, through the composite imaging of the various subdivision layers deriving from the pertaining space parameterization. The inversion problem relating the velocity field construction to travel time data is analyzed within the main theoretical frame of linear solvers: the introduction of a continuous function as Thurber's trilinear interpolator is widely used as effective algorithm to numerically model the variational sequences of data to be processed. The solution of each inversion, iteratively computed on the basis of LSQR methods, is presented in terms of resolution matrix. As far as the covariance matrix with reference to modelling errors, ray-tracing approximation questions are pointed out as essential features of further research on local linear tomography. Ultimately, tomography approaches aimed at identifying natural gas and hydrates in the marine environment whose presence is expressed by "bottom simulating reflector" observed in seismic data is treated in the light of AVO techniques. Heterogeneity of media and amplitude dependence of signals are not explicitly discussed, though advocated.

Introduction

The word *tomography* is a compound noun deriving its meaning from the Greek roots "tomo" (="slice") and "graph" (="image", "trace"). Tomography is actually a computational technique suitable for the observation of the interior of a given structure as perturbative effects interfere with its surface: through multiple reconstructions of various subdivision sections a full dimensional image of the object in question can be obtained. Namely, if we take a slice of a three-dimensional (3-D) object, we get a two-dimensional (2-D) section, and we can reconstruct the 3-D image of the object under investigation by combining many of these 2-D slices. Similarly, a 2-D section can be constructed from multiple one-dimensional (1-D) line integrals that can be measured in an experiment (Lee and Pereira, 1993). In the case the perturbation is represented by a wave, velocity fields become the object of any tomographic analysis whose data are given on travel times terms, under the ray approximation hypothesis for travel waves.

In geophysics, seismic tomography is aimed at describing seismic waves velocity field in a given region, once the arrival times of shock waves have been recorded. With reference to rays approximation models, the velocity field can be evaluated through the integral relation linking rays travel times with the unknown field. In particular, the arrival time T of the seismic wave generated by an earthquake and recorded by a seismic receiver is described by the following equation:

$$T = T_0 + T(\boldsymbol{\alpha}); \quad \text{being } T(\boldsymbol{\alpha}) = \int_{S(\alpha)} \frac{ds}{\boldsymbol{\alpha}} \qquad (0.1)$$

where T_o is the origin time of the event, $T(\boldsymbol{\alpha})$ is the travel time of the ray, S its path from the source (hypocenter) to the receiver (station), dS the path segment, and $\boldsymbol{\alpha}=\alpha(\mathbf{r})=\alpha(x,y,z)$ the unknown velocity field.

Seismic tomography analyses a consistent number of arrival times to provide a 3D description of the field $\boldsymbol{\alpha}=\alpha(x,y,z)$. To be noted that the information content of a single travel time T is distributed along the whole line integral path *S*, whereas the single integral element $\frac{ds}{\boldsymbol{\alpha}}$ should be considered, limiting the angular distribution of crossing raypaths. Unlike medical X-ray tomography, the ray coverage may be not regularly distributed, giving raise to "structure illumination" problems. As far as initial-event locations requirements in order to analyse portions of earth crust, seismic tomography assigns neither the origin time T_o a fixed value nor the ray path S a defining interval, thus implying an *a priori* velocity field introduction for their estimations with consequent non-linear dependence between data and unknowns (Falcone & Beranzone, 1997).

With reference to the arrival times of seismic body wave phases, the travel time T_i of a wave along a path S_i is the integrated slowness $\alpha(\mathbf{r})^{-1}$, already defined in Eq.(0.1):

$$T_i = \int_{S_i} \frac{ds}{\alpha(\mathbf{r})} \qquad \text{for } i=1,\dots,N \tag{0.2}$$

which is often reformulated in terms of $\delta\boldsymbol{\alpha}$ with respect to a starting model $\alpha_0(\mathbf{r})$:

$$\delta T_i = T_i - T_i^0 = \int_{S_i} \frac{ds}{\alpha} - \int_{S_i^0} \frac{ds}{\alpha_0} = \int_{S_i^0} \frac{\delta\alpha(\mathbf{r})}{\alpha_0(r)^2} ds \tag{0.3}$$

Invoking Fermat's principle, second-order terms are neglected and eventually treated as modelling errors. Under this assumption, the integration path S^i approximation by the slightly different path S_i^0 is consistent due to the stationary nature of travel times for small changes away from S_i^0.

In order to discretize $\delta\alpha=\alpha-\alpha_0$, $\alpha(\mathbf{r})$ may be developed from the outset in a finite number of parameters γ_k at some point in the iterative process as:

$$\delta\alpha(\mathbf{r}) = \sum_{k=1}^{M} \gamma_k h_k(\mathbf{r}) \tag{0.4}$$

where γ_k is the weight of an interpolation function h_i in the Earth. The interpolation functions span a basis in the *model space* (Nolet, 1996).

A common parameterization is in terms of *cells*:

$$h_i(\mathbf{r}) \quad \begin{aligned} &= 1 \quad \text{if } \mathbf{r} \text{ is in cell } i \\ &= 0 \quad \text{elsewhere} \end{aligned} \tag{0.5}$$

leading to unphysical solutions in which the Earth velocity changes block-wise.

Subdivision refining and *a posteriori* smoothing images techniques have been adopted to find the optimal resolving power for the solution search. Smooth interpolators, such as the trilinear interpolation proposed by Thurber (1983) are widely used if the cell size exceeds locally the resolution. Dziewonski (1984), Morelli and Dziewonski (1987a) and others prefer to expand the Earth's velocity field into a finite number of fully normalized spherical harmonics with respect to depth functions.

In all cases, the parameterization reduces the tomographic problem to a set of linear equations for the unknown coefficients γ_i:

$$\mathbf{A\gamma=d} \tag{0.6}$$

where

$$A_{ik} = -\int \frac{h_k(\mathbf{r})}{\alpha_0(\mathbf{r})^2} ds \tag{0.7}$$

A is generally considered independent of the model according to Fermat's principle: travel times are stationary with respect to small changes in the raypaths (**A** containing raypaths lengths), thus stating the equivalence of a simple starting model for a true Earth description; second-order terms are treated as modelling errors.

For a very heterogeneous structure the matrix **A** may have to be regarded as model dependent making the tomography problem a non linear one.

<u>3-D Linear Solvers</u>

The Local Earthquake Tomography (LET) technique is a widely used inversion method first introduced by Crosson (1976) and Aki and Lee (1976) further developed by Thurber (1983) and modified by Eberhart-Phillips (1986) to include S-wave arrival times. According to its basic assumptions, the α_p velocity is modelled through the P-wave arrival times, being P-wave travel times determined through three dimensional ray tracing, in the form of approximate reconstruction. This analysis permits the solution to be iteratively determined and, in turn, allows the inclusion of lateral heterogeneity velocity structure in the Earth's crust. The highly

nonlinear problem is first solved iteratively through multiple inversions linearized with respect to a starting estimate of model parameters, in which initial values of P and S wave arrival times are connected to the source location and the medium velocity, and by simultaneously updating hypocentral and model parameters, as further step. The velocity model is parameterized using a 3D mesh, continuously defined by linearly interpolating the values assigned to the grid nodes among the surrounding grid points. The difference of P and S wave travel times T_p-T_s is commonly inverted to obtain the $\frac{\alpha_p}{\alpha_s}$ velocity ratio. Accurate 1D and 3D velocity models are obtained using LET through the implementation of the widely used SIMULPS numerical code for the calculus of α_p and $\frac{\alpha_p}{\alpha_s}$ (Thurber, 1983). This software simultaneously solves a direct problem pertaining the earthquakes locations as unknown variables once the three-dimensional velocity model has been developed from data inversion (inverse problem).
In analytical terms, a linearized inverse problem may be defined as that of solving the matrix equation:

$$\mathbf{Gx=b} \tag{1.1}$$

where $\mathbf{x}\in \Re^n$ denotes a set of model parameters, $\mathbf{b}\in \Re^m$ denotes the observed data and $\mathbf{G}\in \Re^{m\times n}$ is the matrix connecting model parameters and observations. In seismic tomography the **G** matrix is generally both very sparse and large, making the solution computationally intractable. During the last decade the LSQR method has been applied extensively, implying a minimal CPU memory requirement. Through this method the solution of the above equation is found in the sense of L2 norm, i. e. least squares. In terms of the generalized inverse ($\mathbf{G}^{-g}$) of **G** the solution may be written as:

$$\hat{\mathbf{x}} = \mathbf{G}^{-g}\mathbf{Gx} \equiv \mathbf{Rx} \tag{1.2}$$

where $\mathbf{R}\in \Re^{m\times n}$ is the model resolution matrix which may be regarded as a linear filter relating the true and estimated model parameters. Because of the infinite numbers of generalized inverses in a subspace to be least squared, the details of the resolution matrix depend on the chosen generalized inverse. For an estimated model parameter, i. e. a cell velocity x_i , the corresponding diagonal element of **R** (r_{ij}) will be relatively large, whereas off-diagonal elements related to $x_i(r_{ij})$ may be non-neglegible, but well resolved if spatially close to the cell x_i. (Yao et al., 1999)
Using Lanczos lower bidiagonalization by Paige & Saunders (1982a), iterations for the updating of successive approximated x_k are the same as in the conjugate form due to the property of triangular matrices, thus requiring only the most recent columns to be stored, with a sensible reduction of CPU memory. Such algorithms always involve data backprojections through multiplication with the transpose $\mathbf{A^T}$**,** with reference to (1.2), where $\mathbf{A^T=A^{-A}}$. This means that the solution is constructed from rows of the matrix **A** yielding the minimum norm solution, i. e. $\boldsymbol{\gamma}$ is the vector with smallest length among all the ones that solves (0.6).
Probabilistic methods to locate earthquakes from the inverted velocity field have been developed by Lomax et al. (2000, http://www.alomax.net/nlloc) using different algorithms on a global scale (Lomax, 2001) and by Waldhauser and Ellsworth (2000) as relative techniques based on the comparison between the spatial separation of events and the source-station distance in terms of local velocity heterogeneity.
The quality of dataset is a major feature in seismic tomography which may be improved by a comprehensive manual phase repicking both for S and P wave arrival times prior to the location values and the focal parameters search resulting from the algorithms implementation, within the constraints superimposed by the geological knowledge of the area of investigation (Turino & Scafidi, 2008). The reliability of tomographic imaging is also strictly dependent on mapping indicators, such as hit count and derivative weighted sum-DWS in order to assess the spread function, and define the image resolution in the various subdivision layers resulting from the inversion process.

Data interpolation

The main features of the simultaneous inversion method have been set forth by Thurber who introduced the trilinear smooth interpolator, a continuous function as data fitting algorithm, which is the product of linear functions in each space dimension, to calculate the velocity at a given point (x, y, z), being x_i, y_j, z_k the coordinates of the eight grid points surrounding the point (x, y, z):

$$\alpha(x,y,z)=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\alpha(x_i,y_j,z_k)\left(1-\frac{|x-x_i|}{x_2-x_1}\right)\left(1-\frac{|y-y_j|}{y_2-y_1}\right)\left(1-\frac{|z-z_k|}{z_2-z_1}\right) \qquad (2.1)$$

On the basis of Aki and Lee works, the velocity field representing the earth structure is defined at a great number of discrete points and interpolation is performed within the grid boundaries, instead of varying blockwise, as previously stated. The inverse problem which is overdetermined is solved iteratively through approximate ray tracing. The linearized equation for simultaneous inversion relating the arrival time residual r to model parameters may be written as:

$$r=\Delta T_e+\frac{\partial T}{\partial x_e}\Delta x_e+\frac{\partial T}{\partial y_e}\Delta y_e+\frac{\partial T}{\partial z_e}\Delta z_e+\sum_{i=1}^{N}\frac{\partial T}{\partial \alpha_{0e}}\Delta\alpha_{0e} \qquad (2.2)$$

for each observed arrival, where $\Delta T_e, \Delta x_e, \Delta y_e, \Delta z_e$ and $\Delta\alpha_{0e}$ are perturbations to the hypocentral parameters (earthquake origin time and locations) and the partial derivatives of the arrival time are referred to the earthquake coordinates and velocity parameters, in succession. The last differential term can be calculated given the velocity model and the ray path from the earthquake to the observing station (Thurber, 1983).

On algebraic terms, two different solutions satisfying the experimental data set would imply, theoretically, an infinite number of possible other ones. The solution, i. e. the sequence of travel times, is determined iteratively, in compliance with the uniqueness requirement, on the basis of ray tracing schemes. Thurber's approximation identifies circular arcs of various curvature underlying the source to the receiver as ray tracers, in order to estimate the travel time along each significant corresponding ray segment through the three dimensional velocity model. Data are sequentially assimilated as events are processed in agreement with the continuous behaviour of the interpolation function (2.1), whereas lateral evidence, i. e. slabs, is obtained from simple variations of the plane containing the arcs (Fig. 1). To be noted that the perturbative terms of the seismic source included as hypocentral partial derivatives have been calculated on a variational or geometrical argument and eventually neglected upon comparison with the velocity scale variations in the model. This initial perturbation, though explicitly determining the inverse problem (2.2) is actually accounted for as modelling error within the covariance matrix. Damped least squares methods applied to the resolving matrix and related covariance matrix for the error approximation minimize the travel time range, thus finding the required solution. Calculus is based on the symmetrical property of triangular operators, consistently with the trilinearity introduced. Through the velocity field variations, with relative travel times dependence, earthquakes are individually relocated, i. e. iteratively, in the new model, as inversion is simultaneously repeated. Finally, the Fisher's F test states the stopping point of iterations.

From a conceptual view, an improvement of the geometrical tool to linearize, or better, "quasi"-linearize and numerically model the arrival data would be of major interest to trace the variational sequence itself in greater agreement with the initial set, describing heterogeneity of minor grade on a proper dimensional scale.

Ray tracing in heterogeneous media requires solving a non-linear ordinary differential equation or a system of non–linear ordinary differential equations in the variables of interest, upon assumption of initial conditions, i. e. initial position and initial slowness vector. Boundary conditions relative to the rays connection between two points, which is a main geophysical feature when evaluating arrival times, make the solution search more difficult as under the linear assumption. Many formulations exist for the initial value problem, such as the Hamiltonian approach related to the eikonal equation and usual solvers as Runge-Kutta or predictor-corrector schemes suitable for tracing rays, as well as the finite element method. A further step for travel-time tomography with very sophisticated inversion analyses is represented by the diffraction tomography, which considers, in addition, the amplitude among its defining parameters.

Tomography approach for gas hydrate investigations

Gas hydrates, or simply hydrates, also referred as clathrate hydrates are a ubiquitous class of crystalline inclusion compounds consisting of guest molecules trapped in a lattice of polyhedral water cages, physically resembling ice, being water-based solids. Non polar molecules with low molecular weight-typically gases-are trapped as guest elements inside hydrogen bonded water molecules-host molecules, as well as some higher hydrocarbons and freons may act as guest molecules forming hydrates at suitable pressures and temperatures.

Naturally on earth gas hydrates can be found on the seafloor, in ocean sediments, in deep lake sediments, e. g. Lake Baikal, and in permafrost regions. Methane is significantly hosted in natural hydrate deposits in marine sediments, representing such a relevant carbon reservoir to become a dominant factor in estimating unconventional energy resources. Methane hydrate is stable in ocean floor sediments at water depths greater than 300 m, cementing loose sediments in a surface layer several hundreds meters thick, at occurrence. Additionally, conventional gas resources appear to be trapped beneath sedimented methane hydrate layers. Gas-hydrate-cemented strata also act as seals for trapped free gas, providing potential energy reservoirs, and, conversely, their dissociation with subsequent methane release into the atmosphere would be of major environmental impact, being methane a strong greenhouse effect gas.

The occurrence of hydrate is often inferred by bottom simulating reflections (BSRs), reflections events with reversed polarity featuring the seafloor trend, on seismic reflection profiles: the presence of hydrates is commonly expressed by BSRs observed in seismic reflection data. The presence of gas hydrates reduces the effective pore space and permeability of sediments by filling voids in pore water molecules, thereby increasing the acoustic velocity components. If the hydrate layer is underlain by gas or brine-saturarted sediments, seismic velocity drops, and BSR signals across the hydrate/free gas interface due to impedance contrast changes in transition zones may be observed. It is therefore possible to evaluate seismic three-dimensional volumes in terms of hydrate/free gas concentration volumes, by producing informative data about their areal distributions, including the methane contents trapped in the sediments, on a regional scale through Seismic Reflection Tomography (SRT). Bottom Simulated Reflector (BSR) may be actually regarded as a characteristic seismic horizon appearing on marine reflection profiles, marking the thermodynamical parameters of the methane-hydrate stability field.

On analytical terms, the sequence stratigraphic framework within the region of interest requires an additional inversion to quantify the elastic parameters of the BSR interface. Indicators of hydrates layers may in fact be inferred from the velocity model discussed in the previous sections, requiring a first data inversion, and from amplitude variations of the reflected signals with respect to vertical offset changes along the continental margin (further inversion). Amplitude Versus Offset (AVO) methodology is an inversion procedure which is often applied to analyse velocity fields at the separation BSR interface on the basis of single velocity models and relative densities values for the upper and lower layers; geometrical spreading corrections, realignment of the source-receiver array and picking are also controlled by AVO during the amplitude data processing (Grion et al., 1998).

These tomographic techniques based on seismic wave reflections allow the earth's interior to be mapped in order to visualize the sub-surface whose subdivision elements are chosen as box-grids to be illuminated by seismic rays; the individual character of elements is then physically computed and the obtained results are displayed as colour-coded contour mapping of the sub-surface shear planes (Fig. 2; Fig. 3).

Additionally, computed microtomography (CMT) techniques that utilize an intense X-ray synchrotron source to characterize sediment samples, are aimed at producing high-resolution data regarding sediment parameters on grain scale which are indicators of the methane hydrate behaviour on meso-and-macro scales.

Concluding remarks

The Earth is a mechanical body whose behaviour is complex and depends very strictly on the

time scale that one looks at the earth phenomena and on the characteristic length related to this time scale by an appropriate velocity. Seismological data have played an important role for this quantification: the velocity field produced by P and S waves, numerically constructed from the inversion of arrival times determined through three dimensional ray tracing, constitutes the informative data onset suitable at modelling the earth structure involved by seismic activity. The resulting image is obtained through multiple reconstruction of the subdivision layers chosen as box-grids, i. e. elements of the analysis. Global travel tomography aimed at giving the most accurate image of earth crust interiors has been therefore analysed with respect to ray approximation schemes, adopted as effective parameters of inversion processes, focusing on the widely used Thurber's trilinear interpolator, including its geometrical implications for the array dimensionality requirements. Improvements of the algorithm to minimize the focal separation of earthquakes relocation have been emphasized. Inherent analytical questions pertaining the solution search, iteratively evaluated through damped least squares methods, are also pointed out. Ray tracing in heterogeneous media is just mentioned, whereas lateral heterogeneity is implicitly treated on linear terms under the trilinear assumptions already discussed.
Ultimately, on the basis of a renewed interest in the study of natural gas hydrate because of its potential impact on world energy resources, control on seafloor stability, significance as a drilling hazard, and notable influence on climate as a reservoir of a major greenhouse gas, seismic reflection tomography has been described in order to determine the sediments nature underlying continental margins. Essential theoretical issues, including BSR analyses on seismic data profiles in the light of AVO techniques, have been introduced.

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